Collective Excitations of a Bose-Einstein Condensate in an Anharmonic Trap

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We investigate the collective excitations of an one-dimensional Bose-Einstein condensate with repulsive interaction between atoms in a quadratic plus quartic trap. With using variational approaches, the coupled equations of motions for the center-of-mass coordinate of the condensate and its width are derived. Then, two low-energy excitation modes are obtained analytically. The frequency shift induced by the anharmonic distortion, and the collapse and revival of the collective excitations originated from the nonlinear coupling between the two modes, are discussed.

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I. INTRODUCTION

One of the most important characters of an interacting quantum many-body system is its response to external perturbations, where collective excitation modes represent a very effective tool for exploring the role of interactions and testing theoretical schemes. For this reason, measurements of the collective modes in the trapped gases of alkali-metal atoms [1, 2, 3] were carried out soon after the discovery of Bose-Einstein condensates (BECs). For the dilute degenerate gases, the essential physics of the BECs ground state is included in the Gross-Pitaevskii equation (GPE). The nonlinearity, originated from the interatomic interaction, is included in the equation through a mean field term proportional to condensate density. The study about the collective excitations of the condensation has been investigated extensively by using various theoretical methods [4, 5, 6, 7, 8]. The remarkable agreement between measured frequencies and theoretical predictions is one of most important achievements in the investigation of these new systems. Meanwhile, because of the emergence of nonlinearity, a lot of interesting phenomena in the collective excitations of BECs, such as frequency shift [9], mode coupling [9, 10, 11, 12], damping [13, 14], collapse and revival of oscillations [15, 16] and the onset of stochastic motions for strong driving amplitude [17, 18, 19], have been paid much attention.

Recently, the collective dynamics of an one-dimensional trapped ultra-cold Bose gases has attracted considerable attention, since experiments on trapped Bose gases at low temperature have pointed out the occurrence of characteristics 1D feature [20, 21, 22]. In previous works the studies about the collective excitations of BECs in the magnetic traps are mainly limited to the harmonic case. However, in practical situation of experiments, the trap usually is not purely harmonic. With this concern, we study the collective excitations of the BECs in an one-dimensional in a harmonic trap with a quartic distortion. Our aim is to understand how the distortion affects the collective excitations of BECs.

Our study is facilitated by variational approaches. Using a Gaussian trial function, the GPE is transformed into a set of second order ordinary differential equations about some parameters that characterize the condensate wave function. Then we derived the expressions of the two low-energy oscillation modes analytically and the nonlinear coupling between the two modes is revealed. In particular, we find that, a very small anharmonic distortion may cause a significant frequency shift of the excitation modes when the atomic interaction is strong. Finally, we demonstrate that the anharmonic distortion may give rise to the collapse and revival of the collective excitations.

The paper is organized as follows. In Sec.II we derive the governing equations for the center-of-mass coordinate of the condensate and its width. In Sec.III we discuss the collective modes and the frequency shift caused by the anharmonic distortion. In Sec.IV, we demonstrate the collapse and revival of the collective excitations in anharmonic potential. Final section is our conclusion.

II. VARIATIONAL APPROACH AND GOVERNING EQUATIONS

We consider dilute degenerate bosons confined in a cigar shaped trap and assume that the system is far from the Tonks-Girardeau regime [23]. Then, the BECs can be well described by the dimensionless 1D GPE,

$$i\frac{\partial\psi(x,t)}{\partial t} = [-\frac{1}{2}\frac{\partial^2}{\partial x^2} + V(x) + g|\psi(x,t)|^2]\psi(x,t), \quad (1)$$

where the coordinate x is measured in unit of $\sqrt{\hbar/m\omega_x}$ and time is in unit of $1/\omega_x$. ω_x is the x component frequency of the harmonic trap. $\psi(x,t)$ is the macroscopic wave function of the condensate normalized so that $\int|\psi(x)|^2dx = 1$; $g = 4N\pi\alpha_{1d}a_s/\sqrt{\hbar/m\omega_x}$ characterizes the interatomic interaction and is defined in

terms of the s-wave scattering length a_s (below we shall be concerned with repulsive BECs for which $a_s > 0$); $\alpha_{1d} = \int |\varphi(y,z)|^4 dydz / \left(\int |\varphi(y,z)|^2 dydz\right)^{5/2}$ is a coefficient which compensates for the loss of two dimensions [24]. In the above expressions, N is the total number of atoms and $\varphi(y,z)$ is the ground wave function of the lateral dimensions.

The trapping potential we consider takes form

$$V(x) = \frac{1}{2}(x^2 + \lambda x^4). \tag{2}$$

The quartic term in the potential denotes the anharmonicity of the trap. In [25], the authors created such a quartic confinement with a blue-detuned Gaussian laser directed along the axial direction. In their case, the non-rotating condensate was cigar shaped and the strength of the quartic admixture was $\lambda \simeq 10^{-3}$. Here, we regard λ as a controllable parameter and assume that the anharmonicity is weak, i.e., $|\lambda| \ll 1$.

The problem of solving Eq.(1) can be restated as a variational problem corresponding to the minimization of the action related to the Lagrangian density [6]

$$\ell = \frac{i}{2}(\psi\frac{\partial\psi^*}{\partial t} - \psi^*\frac{\partial\psi}{\partial t}) + \frac{1}{2}|\nabla\psi|^2 + V(x)|\psi|^2 + \frac{g}{2}|\psi|^4, \tag{3}$$

where the asterisk denotes a complex conjugate. In order to obtain the dynamics of the condensate in the trapping potential we will find the extremum of Eq.(3) with a set of trial functions. In our case, a natural choice of the trial function is a Gaussian, i.e., we take

$$\psi(x,t) = \eta(t) e^{-\frac{[x-\chi(t)]^2}{2w(t)^2} + ix\alpha(t) + ix^2\beta(t)}. \tag{4}$$

At a given time t, this function defines a Gaussian distribution centered at the position χ with width w. The other variational parameters η, α and β are all real variables. Inserting (4) into (3) we can calculate an grand Lagrangian by integrating the Lagrangian density over whole coordinate space, $L = \int_{-\infty}^{+\infty} \ell dx$. Then, from the Lagrange equations, we obtain the evolution equations for all variational parameters.

The dynamical equations for the center-of-mass and width of the condensate is derived as follows

$$\ddot{\chi} + \chi + 2\lambda\chi^3 + 3\lambda\chi w^2 = 0, \tag{5}$$

$$\ddot{w} + w + 3\lambda w^3 + 6\lambda\chi^2 w = \frac{1}{w^3} + \frac{p}{w^2}, \tag{6}$$

where the effective interaction $p \equiv \frac{g}{\sqrt{2\pi}}$, which comes from the nonlinear interaction between the particles.

The other variational parameters can be obtained from the center coordinate and the width through the equations,

$$\sqrt{\pi}|\eta(t)|^2 w(t) = 1, \tag{7}$$

$$\beta = \frac{\dot{w}}{2w}, \qquad \alpha = \dot{\chi} - \chi\frac{\dot{w}}{w}. \tag{8}$$

The first one corresponds to the normalized condition of the wave function, $\int |\psi(x,t)|^2 dx = 1$. Therefore, once we know the behavior of the center and width of the condensate, we can calculate the evolution of the rest of the parameters, and then completely characterize the dynamics of the condensate.

Comparing the above equations with that of a pure harmonic potential, we find the emergence of two new terms in Eq.(5) and (6), i.e., the third and the fourth terms in the left-side. Obviously, the third term is directly from the distortion of potential. Whereas, the fourth one represents the response of coherent wave to the distortion of potential manifesting a coupling between the motions of center and width.

III. COLLECTIVE MODES AND FREQUENCY SHIFT DUE TO THE ANHARMONIC DISTORTION

When we consider the contribution from the quartic distortion, i.e., $\lambda \neq 0$ in the potential $V(x)$, the nonlinear coupling between the oscillations of the center and width emerges. The equilibrium points of Eq.(5) and Eq.(6) correspond to the stable or unstable stationary states of the condensate. They satisfy following equations,

$$\chi_0 + 2\lambda\chi_0^3 + 3\lambda\chi_0 w_0^2 = 0, \tag{9}$$

$$(1 + 6\lambda\chi_0^2)w_0 + 3\lambda w_0^3 = \frac{1}{w_0^3} + \frac{p}{w_0^2}. \tag{10}$$

There is only one stable equilibrium point for $\lambda > 0$, that is,

$$\chi_0 = 0, \tag{11}$$

$$w_0 + 3\lambda w_0^3 = \frac{1}{w_0^3} + \frac{p}{w_0^2}. \tag{12}$$

For $\lambda < 0$ there are several equilibrium points, one of them is stable and others are unstable. The stable equilibrium point also satisfies Eq.(11) and Eq.(12).

Expanding Eq.(5) and Eq.(6) around the equilibrium points defined by (11) and (12) and making a routine diagonalizing process, we can obtain following frequencies for low-energy excitation modes:

$$\omega_{1,2} = \left[1 + \left(1 \mp \frac{1}{2}\right) 6\lambda w_0^2 + (1 \mp 1)\left(\frac{3}{2w_0^4} + \frac{p}{w_0^3}\right)\right]^{\frac{1}{2}}. \tag{13}$$

which are related to the coupled variation of center and width of the condensate [4, 5, 6, 7, 8]. When $\lambda = 0$, ω_1 corresponds to the dipole oscillation ($m = 1$) characterizing the motion of the center-of-mass, and ω_2 is the frequency of the variation of the condensate width, it is just the low-lying collective mode ($m = 0$).

When the potential is not perfectly harmonic, i.e., $\lambda \neq 0$, the contribution from the quartic term will give rise to a shift on the frequencies. When $\lambda > 0$ the frequency will be blue shifted and when $\lambda < 0$ the frequency will be red shifted. This is true for both single particle and BECs. However, it is interesting that for BECs, the frequency shift is enhanced dramatically by the atomic interaction. The frequencies of two low-energy excitations as the functions of p for above parameters are plotted in Fig.1.

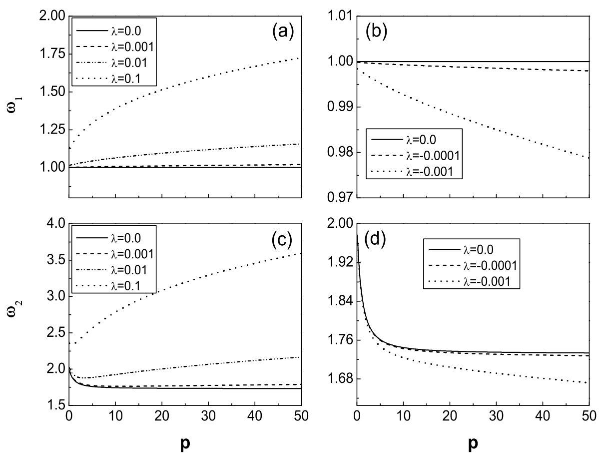

FIG. 1: Frequencies of two low-energy excitations as functions of effective interaction p for different λ.

From Eq.(13), we see that the contribution to the frequency shift comes from the second term in the right-hang, i.e., $\sim \lambda w_0^2$, which is due to the response of coherent wave to the distortion of potential. On the other hand, the width w_0 of the wave function will be broaden by the atom interaction. In Fig. 2. we show this effetc by plotting the dependence of w_0 on λ for different interaction parameters.

From the above discussion, we know, although the anharmonic distortion is very small, the frequency shift maybe large due to magnification effects from the atom interaction. To demonstrate it, in Fig.3 we plot the frequencies of dipole motion of BECs wave-packet for different atom interactions and anharmonic parameters. It is clearly shown that, the atom interaction will give rise to 20% or more shift of frequency when there is only 2% of the anharmonic distortion (see Fig.3, data at $p = 20$). We expect these phenomena can be observed in the future experiment.

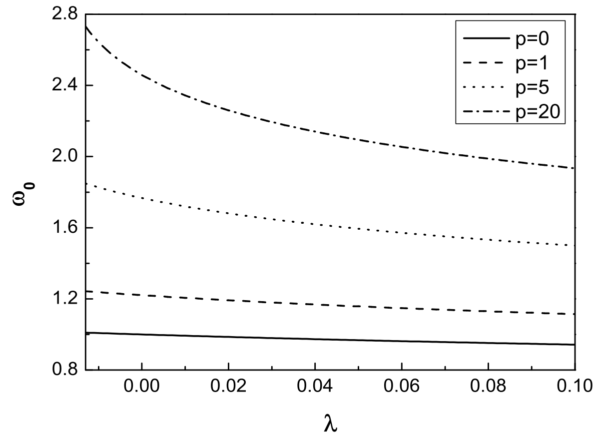

FIG. 2: The equilibrium width w_0 as functions of λ for different effective interaction p.

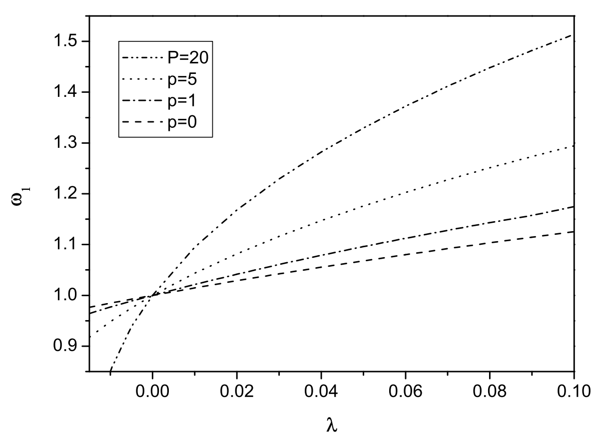

FIG. 3: Frequencies of mass center motions in anharmonic trap as functions of λ for a classical single particle and BEC with different atom interactions p.

IV. COLLAPSE AND REVIVAL OF THE COLLECTIVE EXCITATIONS

Another promising direction is to investigate the collapse and revival of the collective excitations [15, 16], which are directly induced by the nonlinear coupling effect between two oscillation modes. In the previous works, the nonlinear coupling originates from the intrinsic interaction between particles in the system. Here, we show a different mechanism for the nonlinear coupling that is due to the nontrivial anharmonic corrections to the trap.

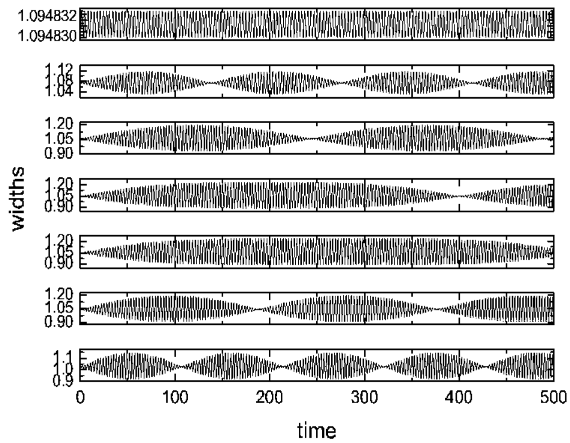

FIG. 4: Variations of the condensate width w for the parameter $p = 0.4$ and the different values of the anharmonic parameter λ. From up to down λ=0, 0.02, 0.04, 0.05, 0.053, 0.06, 0.08.

In our case, the qualitative results can be obtained with making analysis on Eqs.(5) and (6). The oscillation of the center of the condensate couples with the motion of width through the anharmonic parameter λ. It is noted that the coupled motion can be triggered by putting a small shift on the center of the condensate from its equilibrium point. The consequent motions of the width of the condensate excited by the shift are illustrated in Fig. 4 for different anharmonicity parameters. In all the above calculations, initial shift is set as $\triangle\chi = 0.1$. Fig.4 clearly shows the collapse and revival of oscillation patterns for the condensate width with respect to time, induced by the anharmonicity.

Moreover, the change of collapse and revival are not monotonic with increasing λ and the revival period can be effectively controlled by adjusting the anharmonicity parameter. In fact, by linearizing Eqs.(5) and (6) around equilibrium point, we have $\ddot{\chi}'+\omega_1^2\chi' = 0$, and $\ddot{w}'+\omega_2^2 w' = -6\lambda w_0 \chi'^2$. Obviously, the motion of width behaves like a periodically driven oscillator. Since the frequency of the 'external force' is $2\omega_1$, and the intrinsic frequency of the width oscillation is ω_2, so the linear combination of the two frequencies gives the frequency of the collapse and revival, that is, $|2\omega_1 - \omega_2|$. Detailed analysis also suggest that the frequency of the collapse and revival increases monotonically with the nonlinear parameter p, but is not monotonic with increasing λ. It will vanishes for some parameters, e.g. at $\lambda \approx 0.053$ for $p = 0.4$. This is confirmed by our numerical simulations, as in Fig. 4, around $\lambda \approx 0.053$ for $p = 0.4$, the period of collapse and revival becomes much longer.

V. CONCLUSION

We have investigated collective excitations of a Bose-Einstein condensate in an anharmonic trap with using variational approaches and obtained the analytical expressions for the frequencies of the low-energy excitations. It is shown that the two low-energy excitation modes, corresponding to the variations of the center and width of the condensate, couple with each other. The blue-shift and red-shift on the excitation frequency caused by the anharmonic distortion is revealed and found to be more dramatic in the case of strong atomic interaction. Furthermore, the collapse and revival of collective excitations in the anharmonic potential is discussed. We hope our theoretical results will stimulate the experiments in the direction.

Acknowledgments

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[1] D. S. Jin, *et al*, Phys. Rev. Lett. **77** (1996) 420.
[2] M.-O. Mewes, *et al*, Phys. Rev. Lett. **77** (1996) 988.
[3] D. S. Jin, *et al*, Phys. Rev. Lett. **78** (1997) 764.
[4] S. Stringari, Phys. Rev. Lett. **77** (1996) 2360.
[5] M. Edwards, *et al*, Phys. Rev. Lett. **77** (1996) 1671.
[6] V. M. Perez-Garcia, *et al*, Phys. Rev. Lett. **77** (1996) 5320.
[7] F. Dalfovo, *et al*, Phys. Lett. A. **227** (1997) 259.
[8] L. You, W. Hoston, and M. Lewenstein, Phys. Rev. A. **55** (1997) R1581.
[9] F. Dalfovo, C. Minniti, and L. Pitaevskii, Phys. Rev. A. **56** (1997) 4855.
[10] G. Hechenblaikner, *et al*, Phys. Rev. Lett. **85** (2000) 692.
[11] G. Hechenblaikner, *et al*, Phys. Rev. A. **65** (2002) 033612.
[12] Guoxiang Huang, Xin-qi Li, and Jacob Szeftel, Phys. Rev. A **69**, 065601 (2004); Yong-li Ma, Guoxiang Huang, and Bambi Hu, Phys. Rev. A **71**, 043609 (2005).
[13] L. Pitaevskii and S. Stringari, Phys. Lett. A. **235** (1997) 398.
[14] D. M. Stamper-Kurn, *et al*, Phys. Rev. Lett. **81** (1998) 500.
[15] L. Pitaevskii, Phys. Lett. A. **229** (1997) 406.
[16] R. Graham, *et al*, Phys. Rev. A. **57** (1998) 503.
[17] L. Salasnich, Phys. Lett. A. **266** (2000) 187.
[18] Jie Liu, *et al* Phys. Rev. A **73**, 013601 (2006); Jie Liu, *et*

al Phys. Rev. A **72**, 063623 (2005).
[19] Chuanwei Zhang, Jie Liu, Mark G. Raizen, and Qian Niu Phys. Rev. Lett. **92**, 054101 (2004); Jie Liu, Biao Wu, and Qian Niu Phys. Rev. Lett. **90**, 170404 (2003).
[20] A. Görlitz, *et al*, Phys. Rev. Lett. **87** (2001) 130402.
[21] F. Schreck, *et al*, Phys. Rev. Lett. **87** (2001) 080403.
[22] S. Dettmer, *et al*, Phys. Rev. Lett. **87** (2001) 160406.
[23] D.M. Gangardt and G.V. Shlyapnikov, Phys. Rev. Lett. **90** (2003) 010401.
[24] J. E. Williams, in *The Preparation of Topological Modes in a Strongly-Coupled Two-Component Bose-Einstein Condensate*, PhD thesis, Univ. Colorado (1999).
[25] V. Bretin, S. Stockm, Y. Seurin and J. Dalibard, Phys. Rev. lett. **92** (2004) 050403.